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RESEARCH

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# Are rare earth stocks efficient? Novel insights using asymmetric MF-DFA

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## Abstract

This study investigates the weak-form efficiency and asymmetric multifractal scaling behavior of rare earth stock indices in the global, U.S. and Chinese markets during the trade war and the COVID-19 period. We examine the scaling behavior across overall, upward (bullish), and downward (bearish) market states from 2013 to 2021, employing an asymmetric multifractal detrended fluctuation analysis approach. Our findings indicate asymmetric multifractality in U.S. rare earth stock prices, caused by fat tails and long-range correlations. Weak-form price inefficiency and asymmetry in U.S. rare earth stock prices are prominent during market downturns, such as the trade war and COVID-19 periods. Chinese rare earth stocks demonstrate greater efficiency than U.S. and global stocks; thus, the latter markets provide arbitrage opportunities during upward and downward trends.

**Keywords:** Rare earth stocks, Long memory, Efficiency, Asymmetric MF-DFA, COVID-19, Trade war

## Introduction

Rare earth (RE) metals are critical components of clean energy technologies, automobiles, electronic devices, electric vehicles, and various industrial and consumer products. China is the leading supplier of RE materials, accounting for 70.59% of the global RE supply (US Geological Survey 2019). Following the 2015 Paris Agreement, numerous countries committed to pursuing net-zero emissions, increasing demand for clean energy products. RE materials attracted significant attention amid the trade dispute between China and the US, drawing major consumers of RE metals, including the U.S., European Union, and Japan, to explore new mining activities to reduce dependence on China (Schmid 2019).

RE materials are a subset of the commodity market, and investments in this market are on the rise (Reboredo and Ugolini 2020). However, to sustain and grow this investment, a thorough understanding of market behavior and price efficiency is essential (Mokni et al. 2024). Investment in RE materials is expanding with numerous countries actively involved in mining and recycling activities (Rabe et al. 2017). Shahzad et al. (2020) suggest that certain stock market investors concentrate on niche segments due to their information advantage, limiting opportunities for speculators

to secure above-average returns. Academic studies on RE stocks include those by Reboredo and Ugolino (2020), Song et al. (2021), Bouri et al. (2021), Zhou et al. (2022), Kamal and Bouri (2023), and Erer et al. (2023), which examine the connectedness, dependence, and risk transmission mechanisms between RE stocks and other financial markets. Investors are keen to understand not only the risk-return mechanism of stocks but also their behavior under various market conditions. Investor reactions can drive bearish or bullish trends, as their appetite and pricing behavior are not uniform across up and down markets. Market events, whether external (government policies) or internal (splits, rights issues, and warrants), can impact stock market efficiency (Machmuddah et al. 2020). Insights into RE stock efficiency may help policymakers prevent resource misallocation. Therefore, this study aims to understand the price efficiency mechanism of RE stocks during bullish and bearish periods within a semi-strongly efficient market. The study's novelty lies in testing the efficient market hypothesis (EMH) and random walk hypothesis for RE stocks in bullish and bearish markets.

Fama (1970) introduced the EMH to explain how stock prices promptly and accurately adjust to new information. Stock market efficiency can be classified as weak, semi-strong, or strong. Under weak form efficiency, past prices or returns are the best predictors of current market prices, meaning that all publicly available information is already embedded in current prices. Semi-strong efficiency implies that current prices instantly reflect any new (public) information or occurrence, while strong-form efficiency accounts for insider information. Lo (2004) proposed the adaptive market hypothesis (AMH) which explains that market efficiency evolves over time based on the market environment and microstructure. Arbitrage opportunities arise at various timescales, enabling investors to secure above-average returns in an inefficient market. Thus, testing for inefficiency during upside and downside market states has important implications for investors making investment decisions and market regulators developing market regulations. Multifractality is a major characteristic in time series data and can lead to market inefficiency (Cajueiro et al. 2009). An inefficient market is one where exploitable opportunity exists, and there is a chance to gain above-average returns.

Financial markets follow a random-walk process, and asymmetry within these markets remains a significant topic (Farid et al. 2021). Financial markets are cyclical and exhibit bullish and bearish trends, so studying returns requires accounting for multifractal scaling behavior and correlation. However, limited literature is available on the measurement of asymmetric multifractality (Lee et al. 2017). To analyze scaling behavior in time-series data, Alvarez-Ramirez (2010) introduced asymmetric detrended fluctuation analysis (A-DFA). Cao et al. (2013) used asymmetric multifractal detrended fluctuation analysis (A-MF-DFA) to explore asymmetric correlation in the scaling properties of time series data. Asymmetric autocorrelation in financial market time series offers insights into the asymmetric nature of risk. Recent literature on market efficiency includes studies by Chowdhury et al. (2023), who apply asymmetric multifractal cross-correlation analysis; Aslam et al. (2020) and Saâdaoui (2023), who employ MF-DFA analysis; Shen and Chen (2023), who employ asymmetric multifractal spectrum distribution; and Naeem et al. (2023), who apply A-MF-DFA, a refined asymmetric scaling method robust in capturing market trends and offering advantages over symmetric MF-DFA.

In this study, we use A-MF-DFA for several reasons. It discerns multifractal scaling properties in two market trends, bearish and bullish, that are helpful for investors making investment decisions. A-MF-DFA explains inefficiencies that can lead to irrationality in both short positions (excited bullish markets with inflated expectations) and long positions (bearish markets driven by crisis-related fears) (Lee et al. 2017). To evaluate market efficiency, this study uses a market deficiency measure (MDM) based on the hypothesis that a market is efficient if its value is zero (Mensi et al. 2019; Shahzad et al. 2020).

Our study contributes to academic literature in many ways. First, it tests the random walk hypothesis and asymmetric scaling behavior of RE stock indices, which constitute a new asset class for investors. Asymmetric scaling behavior is defined as the segregation of stock indices according to recent market trends (upward and downward), where indices either rise or fall. Second, the multifractal scaling exponents are quantified and compared across RE stock indices to assess their efficiency. Third, we investigate the causes of asymmetric scaling behavior by comparing the outcomes of original series with those of surrogate and shuffled series. Finally, we examine the time-varying weak-form efficiency of RE stocks, which offers a detailed understanding of market regulation for policymakers.

The remainder of the paper is structured as follows: Sect. “Literature review” presents the literature review, Sect. “Data and methodology” describes the data and methodology, Sect. “Results and discussion” presents the results and discussion, and Sect. “Conclusion” concludes the paper.

### Literature review

RE elements are essential minerals with critical roles in advanced technologies. Production of RE elements is largely concentrated in China, creating a global supply risk (Schmid 2019). China’s export policies, especially on RE materials, negatively impact global trade, particularly regarding supply and prices (Mancheri et al. 2019). Geopolitical factors also play a dominant role in shaping China’s RE policies (Wübbecke 2013). Highlighting the importance of RE to China, President Deng Xiaoping stated, “If the Middle East has oil, China has rare earth”. In the future, RE metals could play a role equivalent to oil and other raw commodities in shaping the global political equilibrium (Baldi et al. 2014). RE metals are used in clean energy production, industrial production, electric vehicles, national security applications, and electronic devices (Humphries 2011). To mitigate climate change risks, the world is accelerating demand for green energy to meet sustainable goals set by the Paris Climate Agreement, where 196 countries agreed to limit global temperature rise below 2 °C (Delbeke et al. 2019). Investment in RE elements is projected to grow as the world focuses on clean energy technologies, where RE elements are essential components and serve as an investment vehicle.

Fama (1970) introduces the efficient market hypothesis, positing that a security’s price follows a random walk in an efficient market. Peters (1994) presents the fractal market hypothesis as an alternative, explaining the scaling properties of return distributions. The multifractal nature of financial and empirical data is crucial, as various properties are linked to scale invariance (Stavroyiannis et al. 2019). Thus, studying the scaling behavior of complex financial markets, such as RE stocks, is

a challenge for researchers, academics, and investors. Various multifractal models have been employed to study the fractal features and scaling behaviors of complex nonstationary time series. Multifractality reveals market inefficiency and contradicts the efficient market hypothesis, holding important implications for investors and regulators.

Most studies use MF-DFA, introduced by Kantelhardt et al. (2002), to study multifractality across various markets. Rizvi and Arshad (2017) employed MF-DFA to study the efficiency of the Japanese stock market, which improves over time, showing evidence of multifractal behavior. Similarly, Ikeda (2018) demonstrates the multifractal behavior of the Russian stock market using the MF-DFA approach, indicating a multifractal structure. Ali et al. (2018) use MF-DFA to study the efficiency of conventional and Islamic stock markets, finding that Islamic markets are more efficient. Shahzad et al. (2018) employed MF-DFA to study the efficiency of forex markets revealing that some currencies are more efficient than others. Han et al. (2019) and Ashfaq et al. (2021) examined forex market efficiency by focusing on major currencies using the MF-DFA model and found evidence of multifractal behavior across all sampled currencies. Aloui et al. (2018) explored the European credit sector's efficiency with an MF-DFA model, revealing time-varying efficiency. Investors' reactions and anticipations shift based on positive and negative trends (Mensi et al. 2020), with varying risk appetites between upward and downward trends.

To study asymmetric multifractality, Lee et al. (2018) applied A-MF-DFA to multiple stock indices, finding that while most markets are efficient, inefficiency can vary over time. Similarly, Cao et al. (2013) used A-MF-DFA to study the asymmetric scaling behavior of the Chinese stock market, finding evidence of asymmetric multifractality. Lee et al. (2017) used A-MF-DFA to demonstrate that US stock indices exhibit asymmetric scaling behavior. Mensi et al. (2019) investigated two major cryptocurrencies (Ethereum and Bitcoin) using A-MF-DFA and confirmed their asymmetric multifractality. Naeem et al. (2020) studied the asymmetric efficiency of various cryptocurrency markets using the A-MF-DFA approach and found significant asymmetric multifractality. Kristjanpoller et al. (2020) conducted asymmetric multifractal analysis on equity funds and various cryptocurrencies, demonstrating that asymmetric multifractality exists in the prices of both. Likewise, Ruan et al. (2021) investigated multifractality in Bitcoin futures and the Bitcoin spot market, providing evidence of correlation and fractality. Shahzad et al. (2020) studied the efficiency of clean energy stocks in bullish and bearish markets using A-MF-DFA, whereas Kalamaras et al. (2017) used MF-DFA to air temperature time-series data.

Existing literature suggests that RE elements are fundamental to renewable energy technologies, electronic devices, consumer products, and industrial production. Over the past two decades, technological advancements and renewable energy developments have underscored the significance of RE elements. As the leading global supplier, China holds substantial influence over RE supply, and the U.S.-China trade war has highlighted the topic's importance (Mancheri et al. 2019). Investors seek to understand the dynamics of RE markets, how RE stocks respond to new information, and how market efficiency and inefficiencies evolve over time.

### Data and methodology

In this study, we used daily data for three RE markets (global, U.S., and China), specifically the MVIS Global Clean-Tech Metals ex-China Index<sup>1</sup> (MVGMXC), China Northern Rare Earth Ltd.<sup>2</sup> (the largest RE mining firm in China), and American Rare Earth Ltd.<sup>3</sup> (the largest U.S. RE mining company). The data were obtained from DataStream, covering the period from November 29, 2013, to November 14, 2021. This period was chosen to encompass various significant events, such as the U.S.-China trade war and the COVID-19 pandemic, as well as China’s dominant role in the RE market—holding 36% of the world’s RE reserves (USGS 2020) and 90% of global RE production (Mancheri et al. 2019). Trade tensions between the U.S. and China have further underscored the importance of RE markets, with the U.S. aiming to reduce its reliance on Chinese RE and pursue new mining initiatives, suggesting a potential increase in RE investment in the near future.

In this study, we applied A-MF-DFA proposed by Cao et al. (2013) to assess the weak-form efficiency of RE stocks. Using this method, we explored multifractality in upward and downward trends, which assists investors in making investment decisions. It provides evidence that most time series exhibit similar multifractal properties during both turmoil and normal periods but differ in upward and downward market regimes. There are two sources of multifractality in financial time series: long-term correlations and fat-tailed distributions. To analyze the source of this asymmetry, we used A-MF-DFA, which follows a four-step procedure.

*Step 1:* We have a time series  $\{X(z)\}$ , where  $z = 1, 2, 3, \dots, S$ , and  $S$  is the length of the series. Starting with A-MF-DFA, we calculate the profile of the series  $\{X(z)\}$ , where  $\bar{X} = \sum_{z=1}^S X(z)$ :

$$Y(\nu) = \sum_{z=1}^{\nu} (X(z) - \bar{X}), \quad \nu = 1, 2, \dots, S \tag{1}$$

*Step 2:* In this step, we divide the series  $\{X(z)\}$  and their profiles  $\{Y(z)\}$  into non-overlapping windows,  $S_n = \text{int}(\frac{S}{n})$ , of equal length  $n$ . As  $S$  is not a multiple of the window length  $n$  in most cases, a short portion of the profile remains. Therefore, the procedure is repeated from the other end of the profile, yielding  $2S_n$  windows. Let  $Q_\nu = \{Q_{\nu,t}, t = 1, 2, \dots, n\}$  represent the  $\nu$  time series with length  $n$ , and  $Y_\nu = \{Y_{\nu,t}, t = 1, 2, \dots, n\}$  the integrated profile of the  $\nu$ -th time duration, where  $\nu = 1, 2, 3, \dots, 0.2S_n$ . Therefore, in the  $\nu$ -th segment ( $t = 1, 2, 3, \dots, n$ ), we have:

$$Q_{\nu,t} = X((\nu - 1)n + t) \text{ and } Y_{\nu,t} = Q((\nu - 1)n + t)$$

for  $\nu = 1, 2, 3, 4, \dots, S_n$  and

$$Q_{\nu,t} = X(S - (\nu - S_e)n + t) \text{ and } Y_{\nu,t} = Q(S - (\nu - S_e)n + t)$$

for  $\nu = S_e + 1, \dots, 2S_n$ .

*Step 3:* We calculate the least-square fitted lines  $L_{Q_\nu}(t) = \alpha_{Q_\nu} + \beta_{Q_\nu}t$  and  $L_{Y_\nu}(t) = \alpha_{Y_\nu} + \beta_{Y_\nu}t$  of series  $Q_\nu = \{Q_{\nu,t}, t = 1, 2, \dots, n\}$  and its profile  $Y_\nu = \{Y_{\nu,t}, t = 1, 2, \dots, n\}$  respectively, where  $t$  represents the horizontal coordinates.  $L_{Q_\nu}$  and  $L_{Y_\nu}$  are the linear trends of the  $\nu$ -th segment and its integrated profile,

respectively. The slope of the least square fit line  $\beta_{Q_\nu}$  indicates whether the trend of  $Q_\nu$  is positive or negative.  $L_{Y_\nu}(t)$  is used to detrend the integrated profile series  $Y_\nu$  and estimate the fluctuation function for each segment  $\nu = 1, 2, \dots, 2Sn$ :

$$R_\nu(n) = 1/n \sum_{t=1}^n (Y_{\nu,t} - L_{Y_\nu}(t))^2$$

*Step 4:* When the time series  $X(z)$  exhibits piecewise negative and positive trends, we average the fluctuation function that displays asymmetric cross-correlation scaling properties. The directional trend-wise  $q$ -th order fluctuations are estimated through the following equations:

$$f_q^+(n) = \left( 1/H^+ \sum_{\nu=1}^{2Sn} (\text{sign}(\beta_{Q_\nu}) + 1|2) [f_\nu(n)]^{q/2} \right)^{1/q}$$

$$f_q^-(n) = \left( 1/H^- \sum_{\nu=1}^{2Sn} (-[\text{sign}(\beta_{Q_\nu}) - 1|2]) [f_\nu(n)]^{q/2} \right)^{1/q}$$

where  $H^+ = \sum_{\nu=1}^{2Sn} (\text{sign}(\beta_{Q_\nu}) + 1|2)$  and  $H^- = \sum_{\nu=1}^{2Sn} -(\text{sign}(\beta_{Q_\nu}) - 1|2)$  account for positive and negative trends, respectively. If  $\beta_{Q_\nu} \neq 0$  for all  $\nu=1, 2, \dots, 0.2Sn$ , then  $H^+ + H^- = 2Sn$ .

Traditional MF-DFA uses the average fluctuation function:

$$f_q(n) = \left( 1/2Sn \sum_{\nu=1}^{2Sn} [f_\nu(n)]^{q/2} \right)^{1/q}$$

Thus, if a power-law cross-correlation exists, the power-law relationship is expressed as follows:

$$f_q(n) \sim n^{h(q)}; \quad f_q^+(n) \sim n^{h^+(q)}; \quad f_q^-(n) \sim n^{h^-(q)}$$

In these equations, the overall, downward, and upward trends are represented, respectively, by  $h(q)$ ,  $h^-(q)$  and  $h^+(q)$ . The scaling behavior of stock indices is determined by analyzing the log–log plots of  $f_q(n)$ ,  $f_q^+(n)$  and  $f_q^-(n)$  against  $n$  for each value of  $q$ . Therefore, these equations are presented as follows:

$$\text{Log } f_q(n) = h(q)\text{log}(n) + \text{log}(P1)$$

$$\text{Log } f_q^+(n) = h^+(q)\text{log}(n) + \text{log}(P2)$$

$$\text{Log } f_q^-(n) = h^-(q) \text{log}(n) + \text{log}(P2)$$

The overall, upward, and downward scaling exponents ( $h(q)$ ,  $h^+(q)$ ,  $h^-(q)$ ) are obtained by examining the slope of the log–log plot of  $f_q(n)$  against time scale  $n$  using the Ordinary Least Square (OLS) method. where  $h(q)$  is the generalized Hurst exponent used to observe persistent or long-term memory in a time series. If  $h(2) > 0.5$ , the time series

shows persistent behavior; if  $h(2) < 0.5$ , it shows anti-persistent behavior. The time series follows a random walk or weak-form efficiency if  $h(2) = 0.5$  (Lee et al. 2018). If  $h^+(q) = h^-(q)$  the time series shows symmetric behavior; if  $h^+(q) \neq h^-(q)$  asymmetric correlation, suggesting that correlations vary in upward and downward trends.

The Renyi exponent,  $t(q)$ , which relates to the Hurst exponent  $h(q)$ , is calculated using MF-DFA as:

$$t(q) = qh(q) - 1$$

The Hölder exponent (singularity strength)  $\partial$  and spectrum  $f(\partial)$  are estimated by the equations:

$$\begin{cases} \partial = h(q) + qh'(q), \\ f(\partial) = q[\partial - h(q)] + 1, \end{cases}$$

where  $h'(q)$  explains the derivative of  $h(q)$  with respect to  $q$ , and the spectrum of the Hölder exponent  $f(\partial)$  represents the singularity strength of the series.

Using the MDM approach based on Hurst exponents, this model constructs a series to assess efficiency using the following formula (Mensi et al. 2019):

$$MDM = 1/2 (|h(q_{min}) - 0.5| + |h(q_{max}) - 0.5|)$$

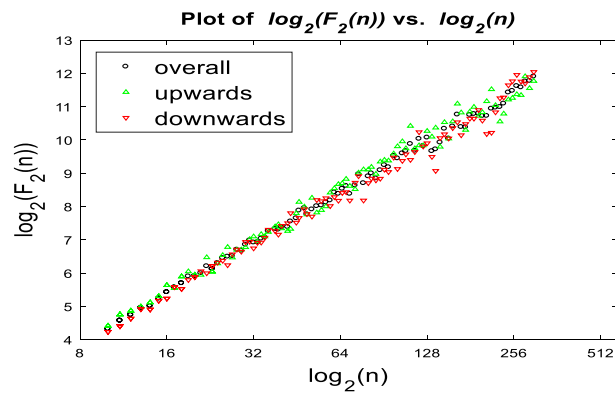
where  $q(min)$  and  $q(max)$  represent the minimum and maximum orders of the fluctuation function (Tiwari et al. 2019; Yao et al. 2021). In our study, the Hurst exponent scales for small and large fluctuations are  $h(-4)$  and  $h(+4)$ , respectively.

## Results and discussion

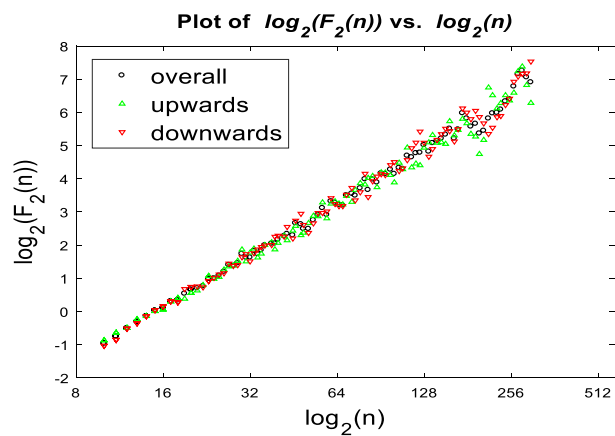
Figure 1 presents a log–log plot of the asymmetric fluctuation function  $F2(n)$  against  $n$  for U.S., China, and global RE stock indices. Interestingly, asymmetric multi-fractality is observed in all three markets. A marked deviation in the upward fluctuation from the downward fluctuation becomes more prominent as  $n$  increases. Long-horizon investors play an important role as they closely monitor asymmetric long-range correlation to outperform the market (Ni et al. 2022). In general, long-term investors are more attuned to RE market behaviors, which may vary significantly between upward and downward trends, allowing them to make informed investment decisions based on their risk appetite. A visual inspection of Fig. 1 shows that symmetric fluctuations are more prominent in the Chinese and global RE markets, whereas a higher deviation is evident in the U.S. market at longer time scales.

Figure 2 illustrates the results for excess asymmetry in the multifractal behavior of the U.S., China, and global RE markets. Graphical evidence demonstrates that excess asymmetry is more pronounced at longer time scales. We calculate this excess asymmetry using the equation  $\Delta hq = H + q - H - q$ . A low or near-zero value represents symmetric behavior in RE markets, whereas higher values indicate more pronounced asymmetric behavior (Erer et al. 2023). Interestingly, we observe excess asymmetry in the Chinese market during upward fluctuations at time scales of 64 to 256. These results suggest that multifractality is stronger in upward trends, indicating inefficiencies in Chinese RE markets during upward trends. Long-term investors consider

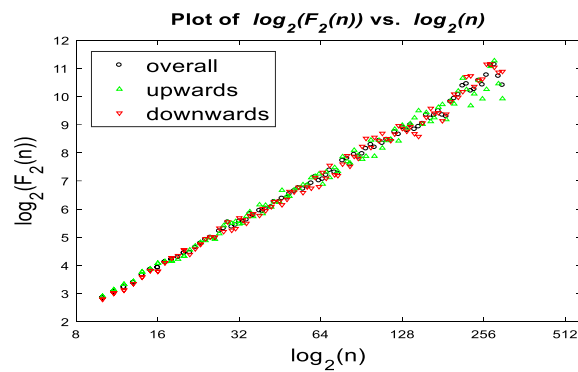




a). China



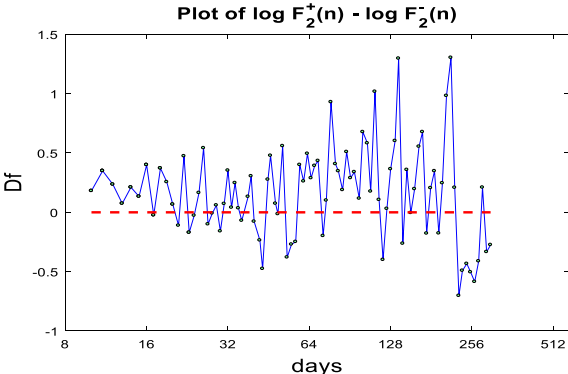
b). USA



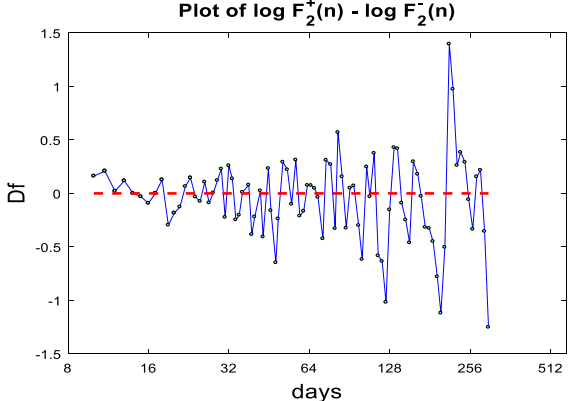
c). Global

**Fig. 1** A-MF-DFA functions:  $F_2(n)$  against time scale  $n$

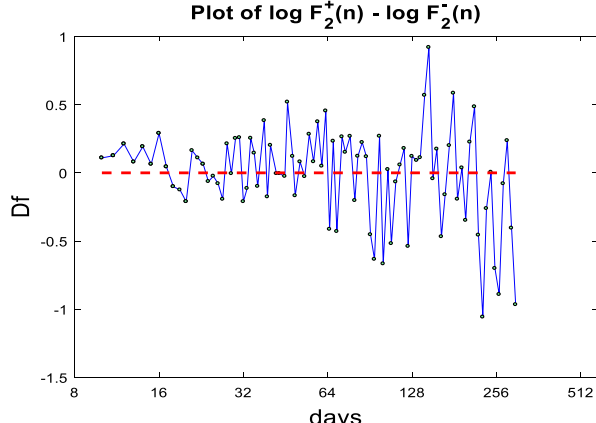
these inefficiencies in upward and downward trends to make informed investment decisions based on their risk and return preferences. Conversely, the U.S. markets exhibit inefficiency particularly during downward trends, with excess asymmetry more visible in downward movements. Interestingly, global RE markets show large positive and negative fluctuations, especially at longer timescales. This implies that



a). China



b). USA



c). Global

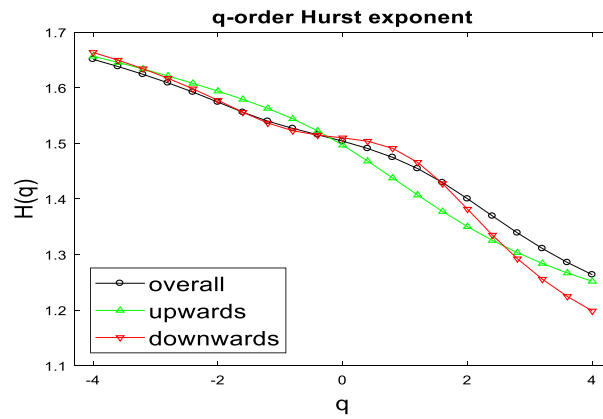
**Fig. 2** Excess asymmetric multifractality

global markets are more efficient in earlier periods, but inefficiency grows over time. Overall, the asymmetric multifractality across markets supports the effectiveness of A-MF-DFA used in this study.

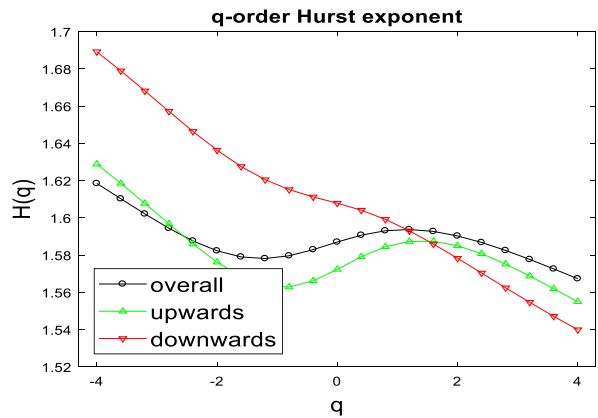
Figure 3 displays the dynamics of the upward Hurst exponent ( $h+q$ ), downward Hurst exponent ( $h-q$ ), and overall Hurst exponent  $h(q)$  for China, U.S., and global RE stocks. Examining the graph reveals that market multifractality varies over scales ranging from  $-4$  to  $+4$ . Chinese RE markets show a more visible gap between the upward and downward Hurst exponents, especially at time scales of  $0$  to  $+4$  with a downward trend from left to right. This suggests that the asymmetric correlation is more pronounced at this scale, enabling investors to achieve above-average returns due to market inefficiency (Mensi et al. 2019; Tiwari et al. 2019). In the U.S. RE markets, the gap between upward and downward trends is more visible between  $-4$  and  $+2$ , diminishing between  $+2$  and  $+4$ , indicating higher market inefficiency lower time scales and relatively efficiency at higher scales. For the global RE market, a higher deviation between upward and downward trends is observed at the lower timescale of  $-4$  to  $+2$ , whereas the market appears more efficient at the higher scales. Overall, the markets display a left-to-right downward trend, indicating an improvement in market efficiency over longer time scales.

Figure 4 presents the multifractal spectra  $f(\alpha)$  against the alpha values for the overall, upward, and downward trends in the Chinese, U.S., and global RE markets. The inverse parabolic shape confirms the existence of asymmetric multifractal behavior. Interestingly, the Chinese RE market shows an upward trend from  $0$  to  $0.5$ , followed by a downward trend. We also observe that the divergence of the upward and downward trends from the overall trend is more distinct. In contrast, the U.S. RE market exhibits an upward trend from  $0.45$  to  $0.6$ , followed by a downward trend. The separation between upward and downward was less apparent in the early stages but became more pronounced beyond  $0.6$ , confirming the presence of asymmetry. The global RE market displays an upward trend from  $0.4$  to  $0.6$  and a downward trend from  $0.6$  to  $1$ , with the divergence between upward and downward trends becoming more visible, specifically between  $0.6$  and  $1$ . The graphical representation suggests that all RE markets experience phases of market inefficiency and efficiency, providing opportunities for long- and short-term investors to make informed investment decisions.

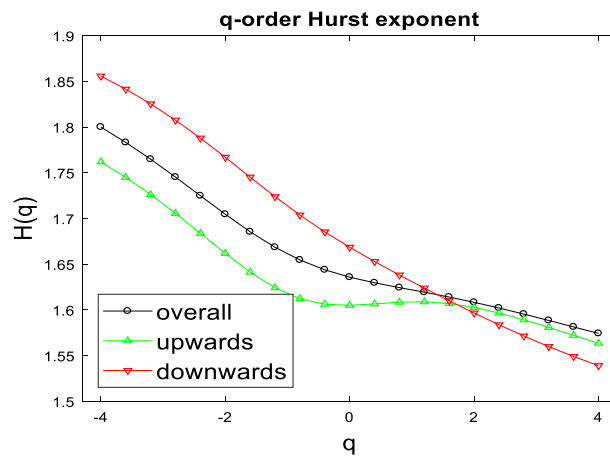
Figure 5 presents a graphical representation of the original, shuffled, and surrogate series for the Chinese, U.S., and global RE stocks. In the case of global RE stocks, the  $\Delta H_{\pm}(q)$  values of the surrogate and shuffled time series are lower than those of the original series, implying that possible sources of asymmetric multifractality are fat-tailed distributions and long-term correlations. However, for the U.S. RE market, the shuffled series has a lower than that of the original series, indicating that the asymmetry originates from a long-range correlation. Conversely, in the Chinese RE market, the values of the shuffled and surrogate series show high volatility, and the shuffled series value is lower than that of both the surrogate and original series, implying that long-term correlation is the source of this multifractality. In China, during the early period, especially from  $-4$  to  $0$ , asymmetric multifractality is less evident; however, from  $0$  to  $4$ , large fluctuations emerge, indicating increased market inefficiency at higher time scales. Consequently, arbitrage opportunities are available in the Chinese RE market at higher orders. In the U.S. market, the disparity is more pronounced from  $-4$  to  $2$ , and less visible from



a). China



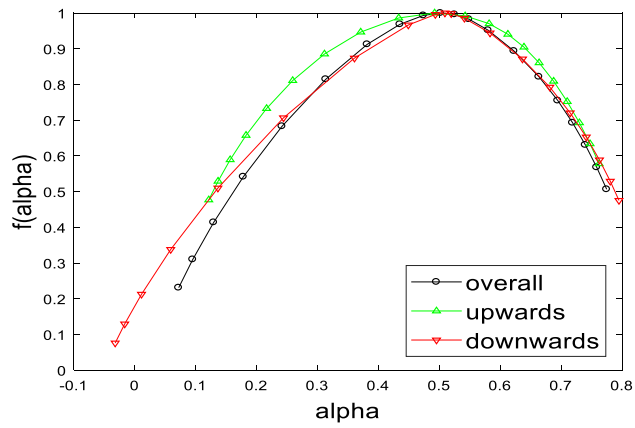
b). USA



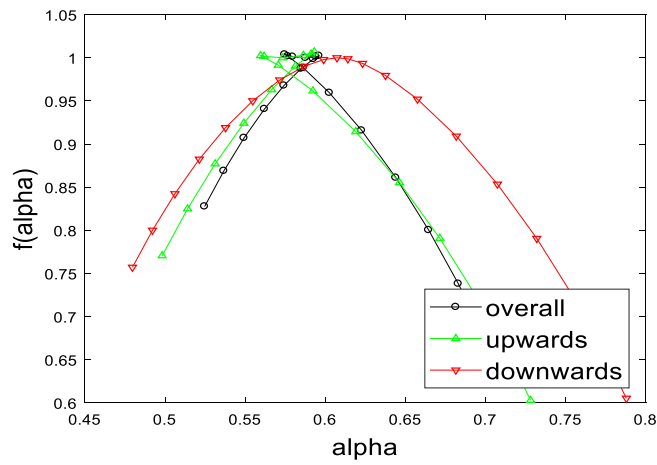
c). Global

**Fig. 3**  $h(q)$ ,  $h+(q)$  and  $h-(q)$  functions of RE stocks against order  $q$

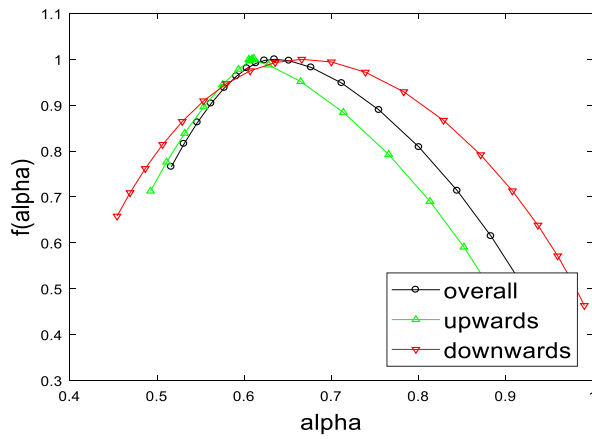
2 to 4, suggesting that the U.S. RE market is less efficient at lower orders and becomes more efficient at higher orders. Additionally, the global market shows reduced efficiency at lower orders, from  $-4$  to 2, and greater efficiency from 2 to 4.



a). China

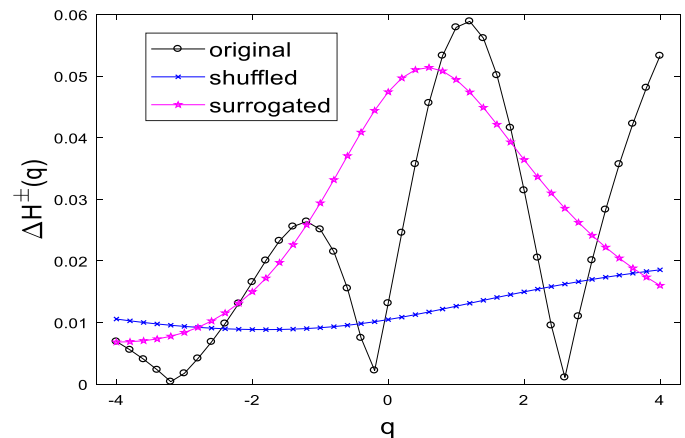


b). USA

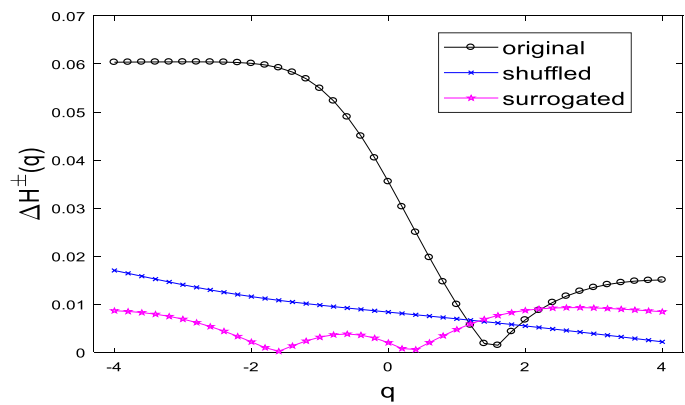


c). Global

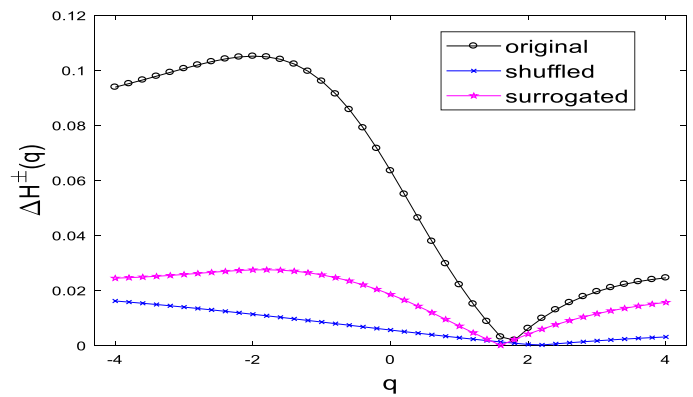
Fig. 4 The asymmetric multifractal spectra  $f(\alpha)$  against  $\alpha$



a). China



b). USA



c). Global

**Fig. 5**  $\Delta H^\pm(q)$  of the original, shuffled and surrogate series

To evaluate the efficiency of the three selected markets (China, the U.S., and global), we used MDM. For the analysis, we assessed the MDM with  $h(-4)$  and  $h(+4)$ , given that markets following a random walk have efficient index values for both small

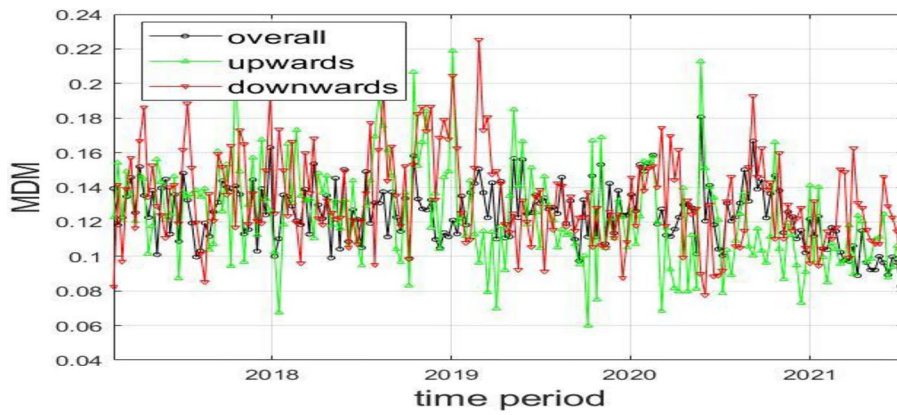
( $h = -4$ ) and large ( $h = +4$ ) fluctuations. The market is considered efficient when the MDM value equals zero.

Figure 6 displays the market inefficiency results for Chinese, U.S., and global RE stocks using the MDM. Examining the plots, China appears more efficient than the U.S. and global indices across overall, upward, and downward trends. In addition, the U.S. and global indices exhibit a distinct patterns for upward and downward trends, whereas China follows a similar pattern, achieving greater efficiency beginning in 2021. Changes in upward and downward trends are significant for portfolio managers and investors. The U.S. and global indices show high volatility prior to COVID-19 (a period often associated with the trade war between the U.S. and China), creating opportunities for arbitrage and abnormal profits. However, post COVID-19, a stricter supply measures from China, the largest exporter of RE elements, reduced opportunities for abnormal profits. Since 2021, with fewer COVID-related trade restrictions, the U.S. and global indices have shown a decline in market efficiency (Hanif et al. 2023).

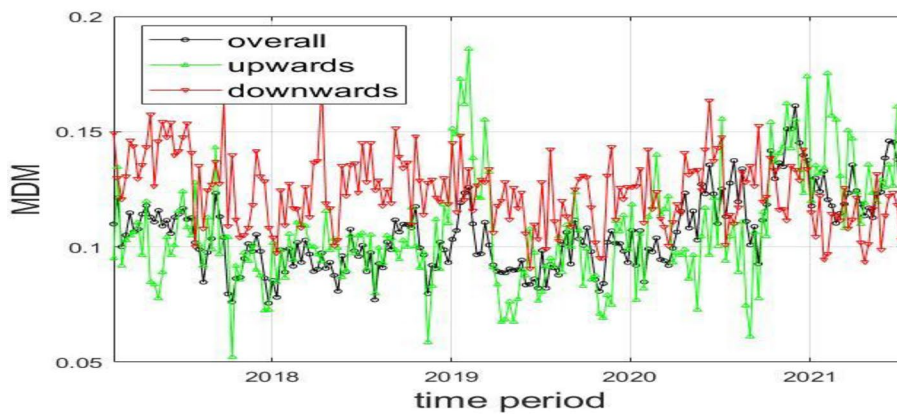
## Conclusion

In this study, we examine the efficiency of RE stocks using A-MF-DFA to understand market behavior during upward and downward trends, as well as during the trade war and COVID-19 periods. First, the Hurst exponent reveals asymmetric multifractality across all RE markets, validating the use of A-MF-DFA. Second, the U.S. RE market displays more significant asymmetric scaling behavior over longer time scales than the Chinese or global RE markets, implying that arbitrage opportunities exist in the U.S., especially at higher timescales. Third, we observe market inefficiency during upward trends in China, suggesting that investors have opportunities to achieve above-average returns during upward trends. Fourth, we identify phases of efficiency and inefficiency at different time scales across various markets; for example, the Chinese market shows market inefficiency in earlier periods, providing arbitrage opportunities for investors. In contrast, the U.S. market is relatively inefficient at higher time scales but exhibits greater efficiencies in the early stages. Finally, possible sources of inefficiency in global markets include long-term correlations and fat-tailed distributions. In both the U.S. and Chinese markets, market inefficiency is observed due to long-term correlations, as the values of the shuffled series are lower than those of the original series.

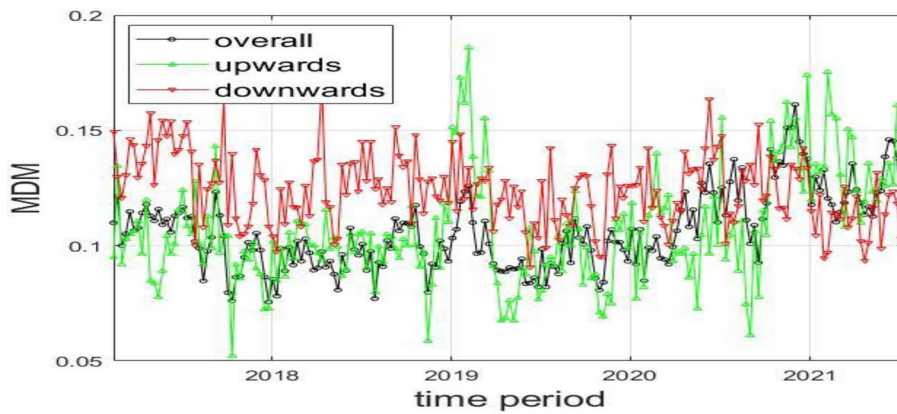
The concept of market efficiency remains controversial in finance literature. Our findings provide insights for policymakers and investors to make investment decisions under varying market conditions, particularly within the niche RE market. Investors can observe how the global, Chinese, and U.S. RE stock markets fluctuate and assess how to hedge risks within the RE market. Furthermore, this study enables regulators to formulate policies for effective risk control in both bullish and bearish market conditions.



a). China



b). USA



c). Global

**Fig. 6** Time-varying dynamics of market efficiency using MDM for  $h(-4)$  and  $h(4)$



**Abbreviations**

RE	Rare earth
AMF-DFA	Asymmetric multifractal detrended fluctuation analysis
EMH	Efficient market hypothesis
AMH	Adaptive market hypothesis

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**Author contributions**

All authors have contributed equally.

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Data will be available on our request.

**Declarations****Competing interests**

We have no conflicts of interest to disclose.

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